6406532844385. ✖ S is a plane in $\mathbb{R}^3$

Question Number : 154 Question Id : 640653845961 Question Type : MSQ
Correct Marks : 2 Max. Selectable Options : 0
Question Label : Multiple Select Question
Select all true statements.
Options :
6406532844386. ✖ S is the empty set.
6406532844387. ✖ S is a singleton set containing the zero vector.
6406532844388. ✔ S = (1, 1, 0) + span{(1, 0, 1)}
6406532844389. ✔ S = (0, 1,−1) + span{(1, 0, 1)}

Sem2 Statistics2

Section Id :	64065360609
Section Number :	7
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	12
Number of Questions to be attempted :	12
Section Marks :	40
Display Number Panel :	Yes
Section Negative Marks :	0
Group All Questions :	No
Enable Mark as Answered Mark for Review and Clear Response :	No
Section Maximum Duration :	0
Section Minimum Duration :	0
Section Time In :	Minutes
Maximum Instruction Time :	0
Sub-Section Number :	1
Sub-Section Id :	640653126024
Question Shuffling Allowed :	No

Question Number : 155 Question Id : 640653845972 Question Type : MCQ
Correct Marks : 0
Question Label : Multiple Choice Question
THIS IS QUESTION PAPER FOR THE SUBJECT "FOUNDATION LEVEL : SEMESTER II: STATISTICS FOR DATA SCIENCE II (COMPUTER BASED EXAM)"

ARE YOU SURE YOU HAVE TO WRITE EXAM FOR THIS SUBJECT?
CROSS CHECK YOUR HALL TICKET TO CONFIRM THE SUBJECTS TO BE WRITTEN.

(IF IT IS NOT THE CORRECT SUBJECT, PLS CHECK THE SECTION AT THE TOP FOR THE SUBJECTS REGISTERED BY YOU)

Options :

6406532844398. ✔ YES

6406532844399. ✖ NO

Question Number : 156 Question Id : 640653845973 Question Type : MCQ

Correct Marks : 0

Question Label : Multiple Choice Question

Discrete random variables:

Distribution	PMF ($f_X(k)$)	CDF ($F_X(x)$)	$E[X]$	$\mathrm{Var}(X)$
$\mathrm{Uniform}(A)$ $A=\{a,a+1,\ldots,b\}$	$\frac{1}{n},\ x=k$ $n=b-a+1$ $k=a,a+1,\ldots,b$	$\begin{cases}0 & x<0\\ \frac{k-a+1}{n} & k\le x<k+1\\ & k=a,a+1,\ldots,b-1,b\\ 1 & x\ge n\end{cases}$	$\frac{a+b}{2}$	$\frac{n^2-1}{12}$
$\mathrm{Bernoulli}(p)$	$\begin{cases}p & x=1\\ 1-p & x=0\end{cases}$	$\begin{cases}0 & x<0\\ 1-p & 0\le x<1\\ 1 & x\ge 1\end{cases}$	p	$p(1-p)$
$\mathrm{Binomial}(n,p)$	${}^nC_k p^k(1-p)^{n-k},$ $k=0,1,\ldots,n$	$\begin{cases}0 & x<0\\ \sum_{i=0}^{k}{}^nC_i p^i(1-p)^{n-i} & k\le x<k+1\\ & k=0,1,\ldots,n\\ 1 & x\ge n\end{cases}$	np	$np(1-p)$
$\mathrm{Geometric}(p)$	$(1-p)^{k-1}p,$ $k=1,\ldots,\infty$	$\begin{cases}0 & x<0\\ 1-(1-p)^k & k\le x<k+1\\ & k=1,\ldots,\infty\end{cases}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
$\mathrm{Poisson}(\lambda)$	$\frac{e^{-\lambda}\lambda^k}{k!},$ $k=0,1,\ldots,\infty$	$\begin{cases}0 & x<0\\ e^{-\lambda}\sum_{i=0}^{k}\frac{\lambda^i}{i!} & k\le x<k+1\\ & k=0,1,\ldots,\infty\end{cases}$	λ	λ

Continuous random variables:

Distribution	PDF ($f_X(k)$)	CDF ($F_X(x)$)	$E[X]$	$\mathrm{Var}(X)$
Uniform$[a,b]$	$\frac{1}{b-a}$, $a \le x \le b$	$\begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x \ge b \end{cases}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exp(λ)	$\lambda e^{-\lambda x}$, $x > 0$	$\begin{cases} 0 & x \le 0 \\ 1-e^{-\lambda x} & x > 0 \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Normal(μ, σ^2)	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$, $-\infty < x < \infty$	No closed form	μ	σ^2
Gamma(α, β)	$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$, $x > 0$		$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$
Beta(α, β)	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ $0 < x < 1$		$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

1. **Markov's inequality:** Let X be a discrete random variable taking non-negative values with a finite mean μ. Then,
$$P(X \ge c) \le \frac{\mu}{c}$$

2. **Chebyshev's inequality:** Let X be a discrete random variable with a finite mean μ and a finite variance σ^2. Then,
$$P(|X - \mu| \ge k\sigma) \le \frac{1}{k^2}$$

3. **Weak Law of Large numbers:** Let $X_1, X_2, \ldots, X_n \sim$ iid X with $E[X] = \mu, \mathrm{Var}(X) = \sigma^2$.
Define sample mean $\overline{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}$. Then,
$$P(|\overline{X} - \mu| > \delta) \le \frac{\sigma^2}{n\delta^2}$$

4. **Using CLT to approximate probability:** Let $X_1, X_2, \ldots, X_n \sim$ iid X with $E[X] = \mu, \mathrm{Var}(X) = \sigma^2$.
Define $Y = X_1 + X_2 + \ldots + X_n$. Then,
$$\frac{Y - n\mu}{\sqrt{n}\sigma} \approx \text{Normal}(0,1).$$

Options :

6406532844400. ✔ Useful Data has been mentioned above.

6406532844401. ✖ This data attachment is just for a reference & not for an evaluation.

Sub-Section Number : 2

Sub-Section Id : 640653126025

Question Shuffling Allowed : Yes

Question Number : 157 Question Id : 640653845974 Question Type : MCQ

Correct Marks : 3

Question Label : Multiple Choice Question

Let $X \sim \text{Exponential}(3)$ with PDF $f_X(x) = 3e^{-3x}$, $x > 0$. Find the PDF of $Y = 2X + 3$.

Options :

6406532844402. ✖

$$f_Y(y) = \begin{cases} 3e^{-3\left(\frac{y-3}{2}\right)} & \text{for } y > 3 \\ 0 & \text{otherwise} \end{cases}$$

6406532844403. ✔

$$f_Y(y) = \begin{cases} \frac{3}{2}e^{-3\left(\frac{y-3}{2}\right)} & \text{for } y > 3 \\ 0 & \text{otherwise} \end{cases}$$

6406532844404. ✖

$$f_Y(y) = \begin{cases} 6e^{-3(y-3)} & \text{for } y > 3 \\ 0 & \text{otherwise} \end{cases}$$

6406532844405. ✖

$$f_Y(y) = \begin{cases} \frac{3}{2}e^{-3\left(\frac{y-3}{2}\right)} & \text{for } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Question Number : 158 Question Id : 640653845976 Question Type : MCQ

Correct Marks : 3

Question Label : Multiple Choice Question

Let $X_1, X_2 \sim$ i.i.d. X, where X is a discrete random variable with the following PMF

$$p_X(x) = \begin{cases} 0.2, & \text{for } x = 0 \\ 0.4, & \text{for } x = 1 \\ 0.4, & \text{for } x = 2 \\ 0, & \text{otherwise.} \end{cases}$$

Define $Y = X_1 + X_2$. Find the MGF of Y.

Options :

6406532844407. ✔ $M_Y(\lambda) = (0.2 + 0.4e^{\lambda} + 0.4e^{2\lambda})^2$

6406532844408. ✖ $M_Y(\lambda) = 0.2 + 0.4e^{\lambda} + 0.4e^{2\lambda}$

6406532844409. ✖ $M_Y(\lambda) = (0.4e^{\lambda} + 0.4e^{2\lambda})^2$

6406532844410. ✖ $M_Y(\lambda) = 0.2 + 0.4e^{3\lambda}$

Question Number : 159 Question Id : 640653845980 Question Type : MCQ
Correct Marks : 3
Question Label : Multiple Choice Question
A political survey is conducted to estimate the proportion of voters in a city who support a particular candidate. The true proportion of voters supporting the candidate is 0.45. A random sample of 500 voters is taken. Using Central limit theorem, what is the approximate probability that the sample proportion is between 0.43 and 0.47?
Options :

6406532844416. ✖ $2F_Z(-0.90)$

6406532844417. ✖ $1 - 2F_Z(0.90)$

6406532844418. ✖ $F_Z(-0.90) - F_Z(0.90)$

6406532844419. ✔ $F_Z(0.90) - F_Z(-0.90)$

Sub-Section Number :	3
Sub-Section Id :	640653126026
Question Shuffling Allowed :	Yes

Question Number : 160 Question Id : 640653845975 Question Type : SA
Correct Marks : 3
Question Label : Short Answer Question
Consider a random variable X that satisfies the following:

$$E[X] = 1,\ E[X^2] = 0,\ E[X^3] = 2$$

Define a new random variable $Y = 4 + X + 3X^2$. Find $\mathrm{Cov}(X, Y)$.

Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Equal
Text Areas : PlainText
Possible Answers :

5

Sub-Section Number : 4

Sub-Section Id : 640653126027

Question Shuffling Allowed : No

Question Id : 640653845977 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix

Question Numbers : (161 to 162)

Question Label : Comprehension

Suppose random variables X and Y are uniformly distributed over the shaded region D, as illustrated in Figure 1:

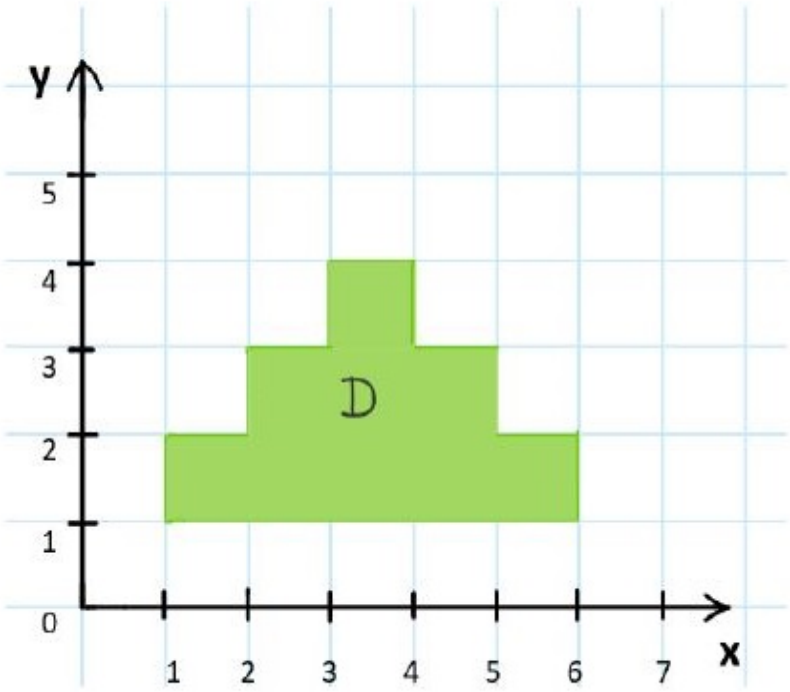

Figure 1: Joint PDF region of X and Y

Based on the above data answer the given subquestions.

Sub questions

Question Number : 161 Question Id : 640653845978 Question Type : SA

Correct Marks : 1

Question Label : Short Answer Question

If their joint PDF is given as

$$f_{XY}(x,y) = \begin{cases} k, & \text{for } (x,y) \in D \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k. Enter the answer correct to two decimal places.

Response Type : Numeric

Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
0.09 to 0.13

Question Number : 162 Question Id : 640653845979 Question Type : MSQ
Correct Marks : 2 Max. Selectable Options : 0
Question Label : Multiple Select Question
Which among the following options are true?
Options :

6406532844412. ✖ $$f_Y(y) = \begin{cases} 2/9, & \text{for } 1 < y \leq 2 \\ 4/9, & \text{for } 2 < y \leq 3 \\ 3/9, & \text{for } 3 < y \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

6406532844413. ✔ $$f_Y(y) = \begin{cases} 5/9, & \text{for } 1 < y \leq 2 \\ 3/9, & \text{for } 2 < y \leq 3 \\ 1/9, & \text{for } 3 < y \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

6406532844414. ✔ $(X \mid Y = a) \sim \text{Uniform}[1, 6]$, for $1 < a < 2$

6406532844415. ✖ $(X \mid Y = a) \sim \text{Uniform}[1, 6]$, for $3 < a < 4$

Sub-Section Number :	5
Sub-Section Id :	640653126028
Question Shuffling Allowed :	No

Question Id : 640653845981 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix
Question Numbers : (163 to 164)
Question Label : Comprehension

Let X and Y have the following joint density function

$$f(x, y) = \begin{cases} 18x^2y^2 & x, y \geq 0, x + y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Based on the above data answer the given subquestions.

Sub questions

Question Number : 163 Question Id : 640653845982 Question Type : MCQ

Correct Marks : 3

Question Label : Multiple Choice Question

Find the conditional distribution $f_{X|Y}(x \mid y)$.

Options :

6406532844420. ✖ $f_{X|Y}(x \mid y) = \frac{3x^2}{(1-y)^3},\ 0 < x < 1$

6406532844421. ✔ $f_{X|Y}(x \mid y) = \frac{3x^2}{(1-y)^3},\ 0 < x < 1-y$

6406532844422. ✖ $f_{X|Y}(x \mid y) = 3x^2(1-y)^3,\ 0 < x < 1$

6406532844423. ✖ $f_{X|Y}(x \mid y) = 3x^2(1-y)^3,\ 0 < x < 1-y$

Question Number : 164 Question Id : 640653845983 Question Type : SA

Correct Marks : 2

Question Label : Short Answer Question

Find the value of $P\left(X < \frac{1}{2} \mid Y = \frac{1}{2}\right)$

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Equal

Text Areas : PlainText

Possible Answers :

1

Question Id : 640653845984 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix

Question Numbers : (165 to 166)

Question Label : Comprehension

A survey reports that the average time spent on social media per day by teenagers is 3 hours, with a standard deviation of 1 hour. A researcher selects a random sample of 64 teenagers.

Based on the above data answer the given subquestions.

Sub questions

Question Number : 165 Question Id : 640653845985 Question Type : SA
Correct Marks : 2
Question Label : Short Answer Question
What is the expected value of the sample mean time spent on social media?
Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Equal
Text Areas : PlainText
Possible Answers :
3

Question Number : 166 Question Id : 640653845986 Question Type : MCQ
Correct Marks : 3
Question Label : Multiple Choice Question
Using Chebyshev's inequality, find the least upper bound on the probability that the sample mean time deviates from the population mean time by more than 0.5 hours.
Options :
6406532844426. ✔ 0.0625
6406532844427. ✖ 0.9375
6406532844428. ✖ 0.031
6406532844429. ✖ 0.015

Question Id : 640653845987 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix
Question Numbers : (167 to 168)
Question Label : Comprehension
The recovery time for a particular surgery in a hospital follows an exponential distribution with a mean of 5 days. A researcher collects data from a sample of 49 patients.

Based on the above data answer the given subquestions.
Sub questions

Question Number : 167 Question Id : 640653845988 Question Type : MCQ
Correct Marks : 2
Question Label : Multiple Choice Question
Using CLT, what will be the approximate distribution of the sample mean recovery time?
Options :
6406532844430. ✖

Normal distribution with mean
5 days and standard deviation 5 days

6406532844431. ✔ Normal distribution with mean
5 days and standard deviation $\frac{5}{7}$ days

6406532844432. ✖ Exponential distribution with mean 5 days

6406532844433. ✖ Exponential distribution with mean $\frac{5}{7}$ days

Question Number : 168 Question Id : 640653845989 Question Type : SA

Correct Marks : 3

Question Label : Short Answer Question

Using CLT, what is the approximate probability that the sample mean recovery time is between 4 and 6 days? Enter the answer correct to two decimal places.

Use the following values of F_Z if required:

- $F_Z(5/7) = 0.76115$
- $F_Z(-5/7) = 0.23885$
- $F_Z(7/5) = 0.91924$
- $F_Z(-7/5) = 0.08076$

Response Type : Numeric

Evaluation Required For SA : Yes

Show Word Count : Yes

Answers Type : Range

Text Areas : PlainText

Possible Answers :

0.82 to 0.86

Question Id : 640653845990 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix

Question Numbers : (169 to 170)

Question Label : Comprehension

Suppose a random variable X is modelled as $X = W + N$, where W is discrete and uniformly distributed in $\{-1, 1\}$ and $N \sim \text{Normal}(0, 1)$, independent of W.

Based on the above data answer the given subquestions.

Sub questions

Question Number : 169 Question Id : 640653845991 Question Type : MCQ
Correct Marks : 3
Question Label : Multiple Choice Question
What is the distribution of *X*?
Options :

6406532844435. ✖ $$f_X(x) = \frac{1}{\sqrt{2\pi}}\left[\exp\left(-\frac{1}{2}(x+1)^2\right) + \exp\left(-\frac{1}{2}(x-1)^2\right)\right]$$

6406532844436. ✖ $$f_X(x) = \frac{1}{2\sqrt{4\pi}}\left[\exp\left(-(x+1)^2/4\right) + \exp\left(-(x-1)^2/4\right)\right]$$

6406532844437. ✔ $$f_X(x) = \frac{1}{2\sqrt{2\pi}}\left[\exp\left(-\frac{1}{2}(x+1)^2\right) + \exp\left(-\frac{1}{2}(x-1)^2\right)\right]$$

6406532844438. ✖ $$f_X(x) = \frac{1}{2\sqrt{2\pi}}e^{-x^2}$$

Question Number : 170 Question Id : 640653845992 Question Type : SA
Correct Marks : 2
Question Label : Short Answer Question
Find the conditional probability $P(W = 1 \mid X = -1)$. Enter the answer correct to two decimal places.
Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
0.10 to 0.14

Question Id : 640653845993 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix
Question Numbers : (171 to 172)
Question Label : Comprehension

Let $X \sim \text{Uniform}[0, 1]$ be a continuous random variable. Define a new random variable $Y = \sqrt{X}$.

Based on the above data answer the given subquestions.

Sub questions

Question Number : 171 Question Id : 640653845994 Question Type : SA
Correct Marks : 2
Question Label : Short Answer Question
Find $E[Y]$. Enter the answer correct to two decimal places.
Response Type : Numeric
Evaluation Required For SA : Yes
Show Word Count : Yes
Answers Type : Range
Text Areas : PlainText
Possible Answers :
0.64 to 0.68

Question Number : 172 Question Id : 640653845995 Question Type : MCQ
Correct Marks : 3
Question Label : Multiple Choice Question
Find the probability density function of Y.
Options :
6406532844441. ✖ $f_Y(y) = y^2, \text{for } y \in [0,1].$

6406532844442. ✔ $f_Y(y) = 2y, \text{for } y \in [0,1].$

6406532844443. ✖ $f_Y(y) = \sqrt{y}, \text{for } y \in [0,1].$

6406532844444. ✖ $f_Y(y) = \frac{y}{2}, \text{for } y \in [0,1/2].$

DBMS

Section Id :	64065360610
Section Number :	8
Section type :	Online
Mandatory or Optional :	Mandatory
Number of Questions :	14
Number of Questions to be attempted :	14
Section Marks :	50
Display Number Panel :	Yes