If the $m \times n$ matrix $B$ is the matrix of $T$ with respect to some basis for $W$ and the standard ordered basis for $\mathbb{R}^2$, then what is $m + n$?

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Equal

**Text Areas :** PlainText

**Possible Answers :**

3

## Sem2 Statistics2

| | |
|---|---|
| **Section Id :** | 64065333914 |
| **Section Number :** | 7 |
| **Section type :** | Online |
| **Mandatory or Optional :** | Mandatory |
| **Number of Questions :** | 12 |
| **Number of Questions to be attempted :** | 12 |
| **Section Marks :** | 40 |
| **Display Number Panel :** | Yes |
| **Group All Questions :** | No |
| **Enable Mark as Answered Mark for Review and Clear Response :** | Yes |
| **Maximum Instruction Time :** | 0 |
| **Sub-Section Number :** | 1 |
| **Sub-Section Id :** | 64065373806 |
| **Question Shuffling Allowed :** | No |
| **Is Section Default? :** | null |

**Question Number : 136 Question Id : 640653520606 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 0**

Question Label : Multiple Choice Question

**THIS IS QUESTION PAPER FOR THE SUBJECT "FOUNDATION LEVEL : SEMESTER 2: STATISTICS FOR DATA SCIENCE 2"**

**ARE YOU SURE YOU HAVE TO WRITE EXAM FOR THIS SUBJECT?**
**CROSS CHECK YOUR HALL TICKET TO CONFIRM THE SUBJECTS TO BE WRITTEN.**

**(IF IT IS NOT THE CORRECT SUBJECT, PLS CHECK THE SECTION AT THE TOP FOR THE SUBJECTS REGISTERED BY YOU)**

**Options :**

6406531735577. ✔ YES

6406531735578. ✖ NO

**Question Number : 137 Question Id : 640653520607 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 0**

Question Label : Multiple Choice Question

Discrete random variables:

| Distribution | PMF ($f_X(k)$) | CDF ($F_X(x)$) | $E[X]$ | $\mathrm{Var}(X)$ |
|---|---|---|---|---|
| Uniform($A$)<br>$A=\{a,a+1,\ldots,b\}$ | $\frac{1}{n}$, $x=k$<br>$n=b-a+1$<br>$k=a,a+1,\ldots,b$ | $\begin{cases}0 & x<0\\ \frac{k-a+1}{n} & k\le x<k+1\\ & k=a,a+1,\ldots,b-1,b\\ 1 & x\ge n\end{cases}$ | $\frac{a+b}{2}$ | $\frac{n^2-1}{12}$ |
| Bernoulli($p$) | $\begin{cases}p & x=1\\ 1-p & x=0\end{cases}$ | $\begin{cases}0 & x<0\\ 1-p & 0\le x<1\\ 1 & x\ge 1\end{cases}$ | $p$ | $p(1-p)$ |
| Binomial($n,p$) | ${}^nC_k p^k(1-p)^{n-k}$,<br>$k=0,1,\ldots,n$ | $\begin{cases}0 & x<0\\ \sum_{i=0}^{k}{}^nC_i p^i(1-p)^{n-i} & k\le x<k+1\\ & k=0,1,\ldots,n\\ 1 & x\ge n\end{cases}$ | $np$ | $np(1-p)$ |
| Geometric($p$) | $(1-p)^{k-1}p$,<br>$k=1,\ldots,\infty$ | $\begin{cases}0 & x<0\\ 1-(1-p)^k & k\le x<k+1\\ & k=1,\ldots,\infty\end{cases}$ | $\frac{1}{p}$ | $\frac{1-p}{p^2}$ |
| Poisson($\lambda$) | $\frac{e^{-\lambda}\lambda^k}{k!}$,<br>$k=0,1,\ldots,\infty$ | $\begin{cases}0 & x<0\\ e^{-\lambda}\sum_{i=0}^{k}\frac{\lambda^i}{i!} & k\le x<k+1\\ & k=0,1,\ldots,\infty\end{cases}$ | $\lambda$ | $\lambda$ |

**Continuous random variables:**

| Distribution | PDF ($f_X(k)$) | CDF ($F_X(x)$) | $E[X]$ | Var($X$) |
|---|---|---|---|---|
| Uniform$[a,b]$ | $\frac{1}{b-a}$, $a \le x \le b$ | $\begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & x \ge b \end{cases}$ | $\frac{a+b}{2}$ | $\frac{(b-a)^2}{12}$ |
| Exp($\lambda$) | $\lambda e^{-\lambda x}$, $x > 0$ | $\begin{cases} 0 & x \le 0 \\ 1-e^{-\lambda x} & x > 0 \end{cases}$ | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ |
| Normal$(\mu, \sigma^2)$ | $\frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$, $-\infty < x < \infty$ | No closed form | $\mu$ | $\sigma^2$ |
| Gamma$(\alpha, \beta)$ | $\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$, $x > 0$ | | $\frac{\alpha}{\beta}$ | $\frac{\alpha}{\beta^2}$ |
| Beta$(\alpha, \beta)$ | $\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ $0 < x < 1$ | | $\frac{\alpha}{\alpha+\beta}$ | $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$ |

1. **Markov's inequality:** Let $X$ be a discrete random variable taking non-negative values with a finite mean $\mu$. Then,
$$P(X \ge c) \le \frac{\mu}{c}$$

2. **Chebyshev's inequality:** Let $X$ be a discrete random variable with a finite mean $\mu$ and a finite variance $\sigma^2$. Then,
$$P(|\ X - \mu\ | \ge k\sigma) \le \frac{1}{k^2}$$

3. **Weak Law of Large numbers:** Let $X_1, X_2, \ldots, X_n \sim$ iid $X$ with $E[X] = \mu, \text{Var}(X) = \sigma^2$.

   Define sample mean $\overline{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}$. Then,
$$P(|\overline{X} - \mu| > \delta) \le \frac{\sigma^2}{n\delta^2}$$

4. **Using CLT to approximate probability:** Let $X_1, X_2, \ldots, X_n \sim$ iid $X$ with $E[X] = \mu, \text{Var}(X) = \sigma^2$.

   Define $Y = X_1 + X_2 + \ldots + X_n$. Then,
$$\frac{Y - n\mu}{\sqrt{n}\sigma} \approx \text{Normal}(0, 1).$$

5. Use the following values of $F_Z$ if required:
   $F_Z(\frac{-5}{6}) = 0.20,\ F_Z(\frac{5}{6}) = 0.80,\ F_Z(2) = 0.977,\ F_Z(-2) = 0.023,\ F_Z(1) = 0.84,$
   $F_Z(\frac{2}{3}) = 0.75,\ F_Z(\frac{-2}{3}) = 0.25,\ F_Z(\frac{5}{2}) = 0.994,\ F_Z(\frac{5}{6}) = 0.797$

6. $\int x^n dx = \frac{x^{n+1}}{n+1}$.

**Options :**

6406531735579. ✔ Useful Data has been mentioned above.

6406531735580. ✖ This data attachment is just for a reference & not for an evaluation.

**Sub-Section Number :** 2

**Sub-Section Id :** 64065373807

**Question Shuffling Allowed :** Yes

**Is Section Default? :** null

**Question Number : 138 Question Id : 640653520611 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3**

Question Label : Multiple Choice Question

Let $(X,Y) \sim \text{Uniform}(D)$, where $D = \{(x,y) : 2 < x+y < 4, x > 0, y > 0\}$. Find $P(Y < 2)$.

**Options :**

6406531735589. ✖ $\frac{1}{3}$

6406531735590. ✔ $\frac{2}{3}$

6406531735591. ✖ $\frac{1}{4}$

6406531735592. ✖ $\frac{3}{4}$

**Question Number : 139 Question Id : 640653520614 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3**

Question Label : Multiple Choice Question

Let $X_1, X_2, \ldots, X_n$ be i.i.d. $X$ with mean $\mu = 0$ and variance $\sigma^2 = 1$. Using Chebyshev's inequality, what should be the minimum value of $n$ such that the probability that the sample mean $\frac{X_1 + X_2 + \cdots + X_n}{n}$ lies between $-0.5$ and $0.5$ is at least $0.95$?

**Options :**

6406531735595. ✖ 40

6406531735596. ✔ 80

6406531735597. ✖ 100

6406531735598. ✖ 95

**Sub-Section Number :** 3

**Sub-Section Id :** 64065373808

**Question Shuffling Allowed :** Yes

**Is Section Default? :** null

**Question Number : 140 Question Id : 640653520612 Question Type : SA Calculator : None**

**Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3**

Question Label : Short Answer Question

The probability density function of a continuous random variable $X$ is given by

$$f(x) = \begin{cases} \frac{6x+1}{10}, & \text{if } 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of $E[X]$. Enter the answer correct to two decimal places.

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Range

**Text Areas :** PlainText

**Possible Answers :**

1.53 to 1.57

**Question Number : 141 Question Id : 640653520613 Question Type : SA Calculator : None**

**Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3**

Question Label : Short Answer Question

Suppose $X_1, X_2, X_3, X_4 \sim$ i.i.d. Bernoulli$\left(\frac{2}{3}\right)$. Define a random variable $Y = 2X_1 + 3X_2 + 4X_3 + 5X_4$. Find Var$(Y)$.

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Equal

**Text Areas :** PlainText

**Possible Answers :**

12

| | |
|---|---|
| **Sub-Section Number :** | 4 |
| **Sub-Section Id :** | 64065373809 |
| **Question Shuffling Allowed :** | No |
| **Is Section Default? :** | null |

**Question Id : 640653520608 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Question Numbers : (142 to 143)**

Question Label : Comprehension

Suppose a fair die is thrown twice independently. Let a random variable *X* denote the number obtained on the first die and a random variable Y denote the number obtained on the second die. Define *Z* =| *7* − X − Y |.

Based on the above data, answer the given subquestions.

**Sub questions**

**Question Number : 142 Question Id : 640653520609 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 1**

Question Label : Multiple Choice Question

Find the range of *Z*.

**Options :**

6406531735581. ✔ $T_Z = \{0, 1, 2, 3, 4, 5\}$

6406531735582. ✖ $T_Z = \{1, 2, 3, 4, 5\}$

6406531735583. ✖ $T_Z = \{1, 2, 3, 4, 5, 6\}$

6406531735584. ✖ $T_Z = \{0, 1, 2, 3, 4, 5, 6\}$

**Question Number : 143 Question Id : 640653520610 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Multiple Choice Question

Find the value of $P(0 < Z < 3)$.

**Options :**

6406531735585. ✖ $\frac{5}{18}$

6406531735586. ✖ $\frac{5}{6}$

6406531735587. ✔ $\frac{1}{2}$

6406531735588. ✖ $\frac{2}{3}$

| | |
|---|---|
| **Sub-Section Number :** | 5 |
| **Sub-Section Id :** | 64065373810 |
| **Question Shuffling Allowed :** | No |
| **Is Section Default? :** | null |

**Question Id : 640653520615 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Question Numbers : (144 to 145)**

Question Label : Comprehension

Kunal throws a dart onto a circular board. Let a random variable *X* denote the distance from the center to the point where the dart hits the board. Suppose the PDF of *X* is

$$f_X(x) = \begin{cases} kx(1-x), & 0 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

Based on the above data, answer the given subquestions.

**Sub questions**

**Question Number : 144 Question Id : 640653520616 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Short Answer Question

Find the value of *k*.

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Equal

**Text Areas :** PlainText

**Possible Answers :**

6

**Question Number : 145 Question Id : 640653520617 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3**

Question Label : Short Answer Question

Find the value of $P(|X - 0.5| \leq 0.25)$. Enter the answer correct to two decimal places.

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Range

**Text Areas :** PlainText

**Possible Answers :**

0.66 to 0.72

**Question Id : 640653520618 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Question Numbers : (146 to 147)**

Question Label : Comprehension

30% of the total candidates in a competitive exam were boys and 70% were girls. The distribution of the marks of the boys is Normal(60,36) and the distribution of the marks of the girls is Normal(55,49).

Based on the above data, answer the given subquestions.

**Sub questions**

**Question Number : 146 Question Id : 640653520619 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3**

Question Label : Multiple Choice Question

Find the PDF of the marks of a candidate chosen uniformly at random.

**Options :**

6406531735601. ✔

$$\frac{1}{20\sqrt{2\pi}}\left(\exp\left(\frac{-(y-60)^2}{72}\right)+2\exp\left(\frac{-(y-55)^2}{98}\right)\right)$$

6406531735602. ✖ $$\frac{1}{20\sqrt{2\pi}}\left(2\exp\left(\frac{-(y-60)^2}{36}\right)+\exp\left(\frac{-(y-55)^2}{49}\right)\right)$$

6406531735603. ✖ $$\frac{7}{60\sqrt{2\pi}}\exp\left(\frac{-(y-60)^2}{72}\right)+\frac{3}{70\sqrt{2\pi}}\exp\left(\frac{-(y-55)^2}{98}\right)$$

6406531735604. ✖ $$\frac{1}{2\sqrt{2\pi}}\left(\frac{1}{6}\exp\left(\frac{-(y-60)^2}{72}\right)+\frac{1}{7}\exp\left(\frac{-(y-55)^2}{98}\right)\right)$$

**Question Number : 147 Question Id : 640653520620 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Multiple Choice Question

If a randomly selected candidate got 60 marks, what is the probability that the selected candidate is a boy?

**Options :**

6406531735605. ✖ $$\frac{2}{2+\exp\left(\frac{-25}{49}\right)}$$

6406531735606. ✖ $$\frac{2}{2+\exp\left(\frac{-25}{98}\right)}$$

6406531735607. ✖ $$\frac{1}{20\sqrt{2\pi}}$$

6406531735608. ✔

$$\frac{1}{1+2\exp\left(\frac{-25}{98}\right)}$$

**Question Id : 640653520621 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Question Numbers : (148 to 149)**

Question Label : Comprehension

At a particular petrol pump, petrol is stocked in a bulk tank each week. Let a random variable *X* denote the proportion of the tank's capacity that is stocked in a given week, and let *Y* denote the proportion of the tank's capacity that is sold in the same week. The petrol pump cannot sell more than what was stocked in a given week. Assume the joint density function of *X* and *Y* is given by

$$f_{XY}(x,y)=\begin{cases}cxy, & \text{if } 0\le y\le x\le 2,\\ 0, & \text{otherwise.}\end{cases}$$

Based on the above data, answer the given subquestions.

**Sub questions**

**Question Number : 148 Question Id : 640653520622 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Short Answer Question

Find the value of *c*. Enter the answer correct to one decimal place.

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Range

**Text Areas :** PlainText

**Possible Answers :**

0.48 to 0.52

**Question Number : 149 Question Id : 640653520623 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3**

Question Label : Short Answer Question

Find the probability that the amount of petrol sold in a given week is less than half the amount stocked in that week. Enter the answer correct to two decimal places.

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Range

**Text Areas :** PlainText

**Possible Answers :**

0.23 to 0.27

**Question Id : 640653520624 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Question Numbers : (150 to 151)**

Question Label : Comprehension

In a manufacturing company, each machine produces 600 bottles daily. If a bottle is selected uniformly at random, then the probability that the bottle is of good quality is 60%.

Based on the above data, answer the given subquestions.

**Sub questions**

**Question Number : 150 Question Id : 640653520625 Question Type : MCQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Multiple Choice Question

Suppose a sample of 20 bottles is selected for a quality inspection. Let a random variable *X* denote the total number of bottles that are of bad quality in the selected sample. Which of the following is true?

**Options :**

6406531735611. ✖ $X \sim \text{Binomial}(600, 0.6)$

6406531735612. ✖ $X \sim \text{Binomial}(20, 0.6)$

6406531735613. ✔ $X \sim \text{Binomial}(20, 0.4)$

6406531735614. ✖ $X \sim \text{Binomial}(600, 0.4)$

**Question Number : 151 Question Id : 640653520626 Question Type : SA Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3**

Question Label : Short Answer Question

Using the Central Limit Theorem, find the approximate probability that a machine will produce more than 370 bottles that are of good quality on a particular day. Enter the answer correct to two decimal places.

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Range

**Text Areas :** PlainText

**Possible Answers :**

0.17 to 0.23

**Question Id : 640653520627 Question Type : COMPREHENSION Sub Question Shuffling Allowed : No Group Comprehension Questions : No Question Pattern Type : NonMatrix Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Question Numbers : (152 to 153)**

Question Label : Comprehension

Let $X$ be a random variable with PMF as follows:

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $f_X(x)$ | 1/4 | 1/2 | 1/4 |

Suppose $X_1, X_2 \sim$ i.i.d. $X$. Define a random variable $Y = X_1 + X_2$.

Based on the above data, answer the given subquestions.

**Sub questions**

**Question Number : 152 Question Id : 640653520628 Question Type : MSQ Is Question Mandatory : No Calculator : None Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 3 Selectable Option : 0**

Question Label : Multiple Select Question

Which of the following option*(s)* is (are) true about the moment generating function of the random variable *Y* ?

**Options :**

6406531735616. ✔ $M_Y(\lambda) = M_{X_1}(\lambda) \times M_{X_2}(\lambda)$

6406531735617. ✖ $M_Y(\lambda) = M_{X_1}(\lambda) + M_{X_2}(\lambda)$

6406531735618. ✖ $M_Y(\lambda) = \frac{1}{2} + e^{\lambda} + \frac{1}{2}e^{2\lambda}$

6406531735619. ✖ $M_Y(\lambda) = \frac{1}{16} + \frac{1}{8}e^{\lambda} + \frac{3}{8}e^{2\lambda} + \frac{3}{8}e^{3\lambda} + \frac{1}{16}e^{4\lambda}$

6406531735620. ✔ $$M_Y(\lambda) = \frac{1}{16} + \frac{1}{4}e^{\lambda} + \frac{3}{8}e^{2\lambda} + \frac{1}{4}e^{3\lambda} + \frac{1}{16}e^{4\lambda}$$

**Question Number : 153 Question Id : 640653520629 Question Type : SA Calculator : None**

**Response Time : N.A Think Time : N.A Minimum Instruction Time : 0**

**Correct Marks : 2**

Question Label : Short Answer Question

Find the expected value of $Y$ .

**Response Type :** Numeric

**Evaluation Required For SA :** Yes

**Show Word Count :** Yes

**Answers Type :** Equal

**Text Areas :** PlainText

**Possible Answers :**

2